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On James Tenney's *Arbor Vitae* for String Quartet

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Arbor Vitae *(2006) for string quartet is James Tenney's last work. The title presents the image of a tree as the central metaphor for the work's harmonic structure, which is similar to the way tree branches emanate from other branches. The harmonic form of* Arbor Vitae *is a series of related tonalities modulating through a richly populated, extended just intonation pitch space. The piece explores the progression of single tonalities expanding into multiple tonalities. This article examines the inner workings of* Arbor Vitae *and the musical result. It documents the algorithm Tenney defined to generate the piece and provides a history of the piece.*

Keywords: James Tenney; Arbor Vitae*; Just Intonation; Algorithmic Composition; Computer Music; Harmonic Distance*

Introduction

Arbor Vitae (2006), James Tenney's last work, is a culmination of many of his ideas, and understanding the piece can provide a certain comprehensive perspective on Tenney's work. The title presents the image of a tree as the central metaphor for the work's harmonic structure. The term 'arbor vitae' ('tree of life') appears in Cage's *Empty Words* (Cage, 1974). Considering Tenney's interest in and scholarship on Cage's work, this is quite possibly an intentional allusion.

Ideas in *Arbor Vitae* Tenney had implemented in the past include deriving a pitch set from the harmonic series, and on the macro-level, defining a single gestalt formally articulated by several parameters. That form is perhaps best understood in *Arbor Vitae* as a number of abstract 'swells' (see Polansky (1983), on the 'swell idea'): an expanding then contracting pitch range, a crescendo/plateau/decrescendo dynamic swell and an increasing then decreasing temporal density (Tenney, 1988 [1964]). All of these 'swells' are applied simultaneously. In this sense, *Arbor Vitae* resembles many of Tenney's earlier works such as *Diapason* (1996), the *Spectrum* series (1995 – 2001), the

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computer pieces from Bell Labs (1961 – 1964), the *Swell* pieces (1967 – 1971) and a great many others.

Arbor Vitae also introduces radical extensions of earlier ideas. For example, pitches are derived from harmonics up to the 1331st partial, whereas earlier works tend to use the first 64 partials or fewer. In addition, the evolving harmonies are determined by a complex, time-variant probabilistic scheme typical of Tenney's work, but executed in *Arbor Vitae* in a unique way.

Arbor Vitae explores the progression of single tonalities expanding into multiple tonalities. The harmonic structure of the piece is similar to the way tree branches emanate from other branches. We can see a tree—or this piece—at various levels of remove, various perspectives, from the detail of the outermost branches to the entire tree as a single, simple, unified form. Despite the compositional rigor of *Arbor Vitae*, hearing it performed is essential to understanding it. This technical description is meant to complement that experience.

The Sound of *Arbor Vitae*

Arbor Vitae was completely generated by an algorithm defined by Tenney. Before discussing the algorithm, I will give a general description of the sound of the piece. *Arbor Vitae* is thirteen minutes long. The harmonic form of the piece is a series of related tonalities (with overlapping transitions) modulating through a richly populated, extended just intonation pitch space. The harmonic trajectory extends quite far, in complex relationship to B-flat, the fundamental for the entire piece. The harmonic space (Tenney, 1983), which at the beginning is populated exclusively with pitches derived from higher primes, higher exponents and compound numbers, expands inwards toward the fundamental. As the piece progresses, newly introduced pitches tend to be more closely related to B-flat, or to use Tenney's own terminology, tend to progress from having greater harmonic distances to lesser harmonic distances, in relation to the fundamental. However, pitches distantly related to the fundamental sound throughout the entire duration of the piece, but the ratio of distantly related pitches to closely related ones gets smaller and smaller. Thus, the harmonies comprising *all sounding pitches* actually become more complex throughout much of the piece.

Arbor Vitae begins with soft, long, sustained tones in the uppermost part of the string quartet's range, primarily played as artificial harmonics or with high stopped strings. The relationships between sounding pitches in the beginning are mostly simple intervals such as just thirds, fifths and minor sevenths, even though at this point they are distantly related to the fundamental. As the piece progresses, harmonic shifts occur more frequently, the tone durations decrease and there is a continual crescendo. The pitch range gradually expands as well: the upper limit of the range remains constant while the lower limit descends. After 1′40″, pitches begin to imply multiple tonalities simultaneously. The lower limit of the pitch range continues to descend, resulting in more and more tones played with stopped strings or as natural harmonics. The sound of the ensemble becomes more robust, active, louder and less ethereal. Past 4′30″, tones continue to shorten and get louder. The harmonic space continues to expand inwards, but does not yet include the B-flat. Thus, the rapidly shifting harmonies imply more and more tonalities at once.

Seven minutes into the piece, the pitch range begins to contract. The upper limit of the range, which has remained constant until this point, begins to descend. By 8′00″, the first sounding of the B-flat has occurred and the piece achieves a maximal intensity: the dynamic level reaches a loud plateau and tone durations are (on average) the shortest in the piece. By this time, tones are primarily realized with stopped strings or as natural harmonics. The upper limit of the pitch range continues to descend until ten minutes into the piece. During this time, the activity shifts to the lower instruments (viola and cello), which play short tones while the violins hold relatively longer ones at higher pitches. By this point, the harmonies are more clearly related to the B-flat fundamental. From 10′00″ until the end, the upper and lower limits of the pitch range ascend back to the uppermost part of the string quartet's range. Tone durations lengthen and the dynamic level softens. The timbre returns to the more ethereal texture of the beginning. *Arbor Vitae* ends as it begins, with one exception. At the beginning, pitches are all distantly related to the fundamental. At the end, all available pitches are active. At the beginning, only the outermost branches are heard; by the end, we hear the entire 'tree'.

Harmonic Overview

In *Arbor Vitae*, a 'root' is some harmonic (integer multiple) of B-flat$_1$,[1] (approximately 58.27 Hertz). The sounding pitch classes are derived from harmonics of successively chosen roots, called 'branches'—that is, pitches are derived from harmonics of harmonics. Thus, every pitch in the piece is harmonically related to B-flat. Branches are calculated by one or more multiplications on the chosen root by 1, 3, 5, 7 or 11.[2] For example, if a branch is 105 and the chosen root is 3, the branch was calculated by $3 \cdot 5 \cdot 7$ and the sounding pitch class is G -43¢.[3] Sometimes the chosen root is simply multiplied by 1 so that the branch equals the chosen root—that is, the branch is the 1st harmonic of the chosen root. Roots and branches lie on what Tenney called 'diagonals'. A diagonal is a set of harmonics (in relation to the fundamental) that have the same number of prime factors (not necessarily distinct). The fundamental is the only harmonic on the 1st diagonal. The 2nd, 3rd and 4th diagonals comprise harmonics with 1, 2 and 3 prime factors, respectively.

Figure 1[4] shows the relationships between the harmonics of the fundamental that derive the pitches in *Arbor Vitae*. As mentioned earlier, Tenney considered these relationships as a function of harmonic distance in harmonic space. In *Arbor Vitae*, harmonic distance in relation to the fundamental is correlated to diagonal. Pitches are displaced by octaves to bring them into the string quartet's instrumental range. For example, only octave displacements of the fundamental and high partials occur.

Early in the piece, roots lie on the 3rd diagonal and branches lie on the 4th diagonal. During this time, there is one tonality at any moment. In Figure 2

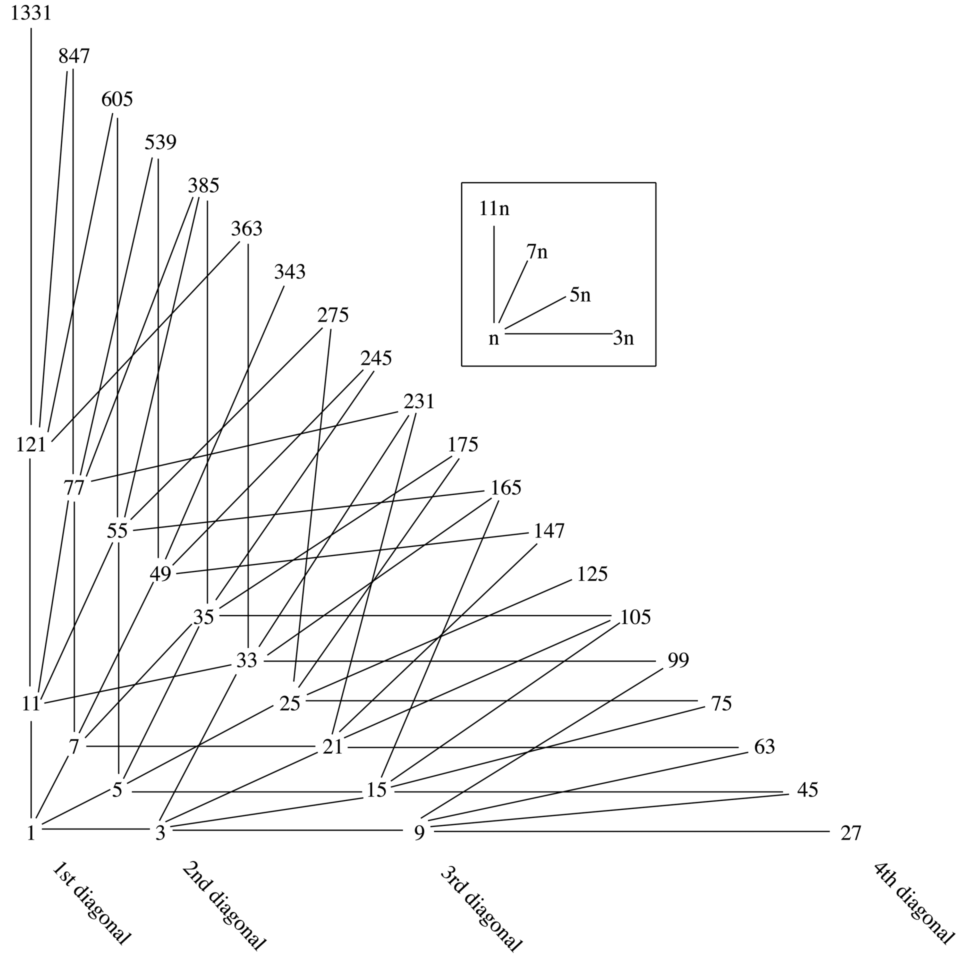

Figure 1 Harmonic structure. All graphs are transcribed from Tenney's original notes.

(25″ – 40″), the harmony comprises tones of the same pitch class as partials (branches) of the currently chosen root, which is the 15th harmonic of the fundamental (even though members of the 15th harmonic's pitch class, approximately A −12¢, are not sounding). For example, the C-sharp −25¢[5] in the 1st violin part is a just major third (derived from the 5th partial) above A −12¢, which is a just major seventh (derived from the 15th partial) above B-flat. In Figure 2, a cents deviation is written directly above each note. The top number next to the arrow extending from each note indicates the partial in relation to the root and the bottom number indicates the partial in relation to the fundamental.

Figure 3 (1′30″ – 1′40″) shows a root transition from the 49th partial to the 9th partial of the fundamental. Both roots lie on the 3rd diagonal because 49 and 9 are

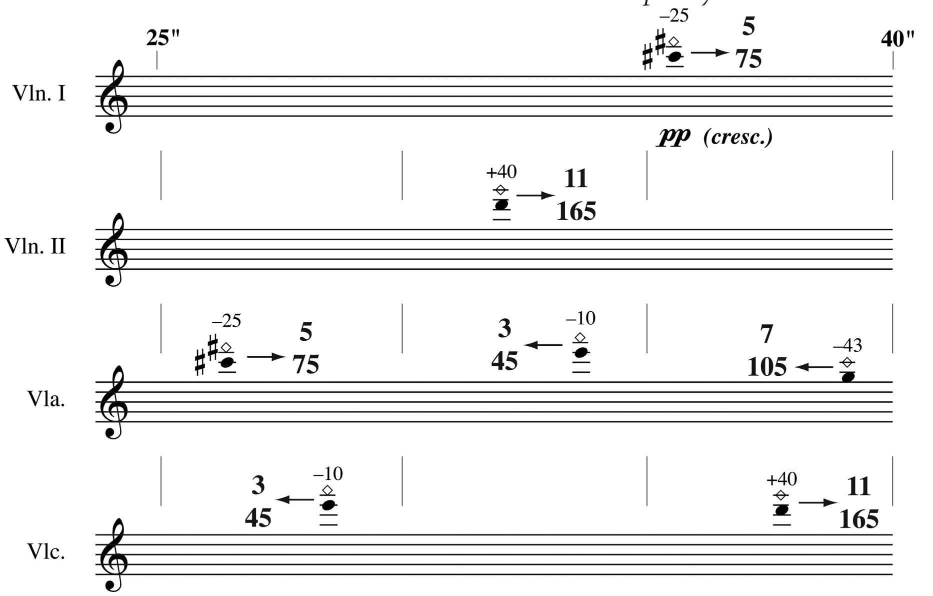

Figure 2 Score excerpt (25″ – 40″). © 2006 James Tenney. All score excerpts from *Arbor Vitae* used by permission. Published by Frog Peak Music (http://www.frogpeak.org).

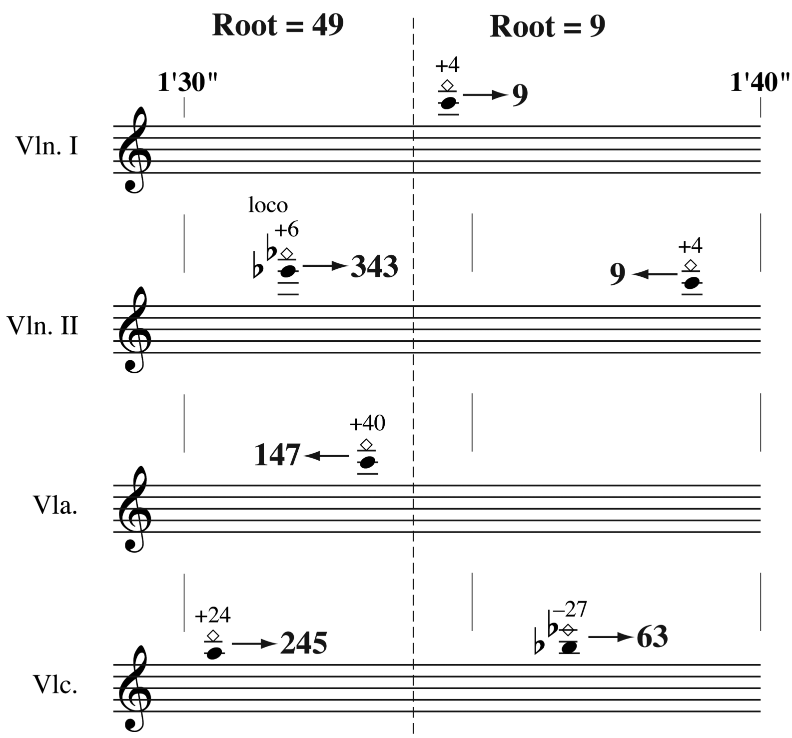

Figure 3 Score excerpt (1′30″ – 1′40″).

7 and 3 squared, respectively. The number next to the arrow extending from each note indicates the partial in relation to the fundamental. Note that the numbers to the left of the dashed line are multiples of 49, and the numbers to the right of the dashed line are multiples of 9. In this example, there are branches that equal the root, 9.

As the piece progresses, roots lie on the 2nd, then 1st diagonal. This results in increasingly polytonal harmonies since sets of branches may share a common divisor that is not the root. Also, pitches may simultaneously imply more than one tonality since a branch may be a compound integer in relation to the root. For example, the 45th harmonic of B-flat (E −10¢) is the 5th partial of the 9th harmonic (C +4¢), and the 3rd partial of the 15th harmonic (A −12¢).

Figures 4 and 5 (5′40″ – 5′50″ and 6′20″ – 6′30″, respectively) show harmonies consisting of multiple tonalities. Lines connect pitches in the same tonality. The top number next to each note indicates the harmonic (in reference to the fundamental) of the same pitch class as the written note. The bottom left number indicates the harmonic of B-flat that is the greatest common divisor of the notes connected by lines. The bottom right number indicates the harmonic (in relation to the harmonic indicated by the bottom left number, the greatest common divisor) of the same pitch class as the written note. Note that multiplying the

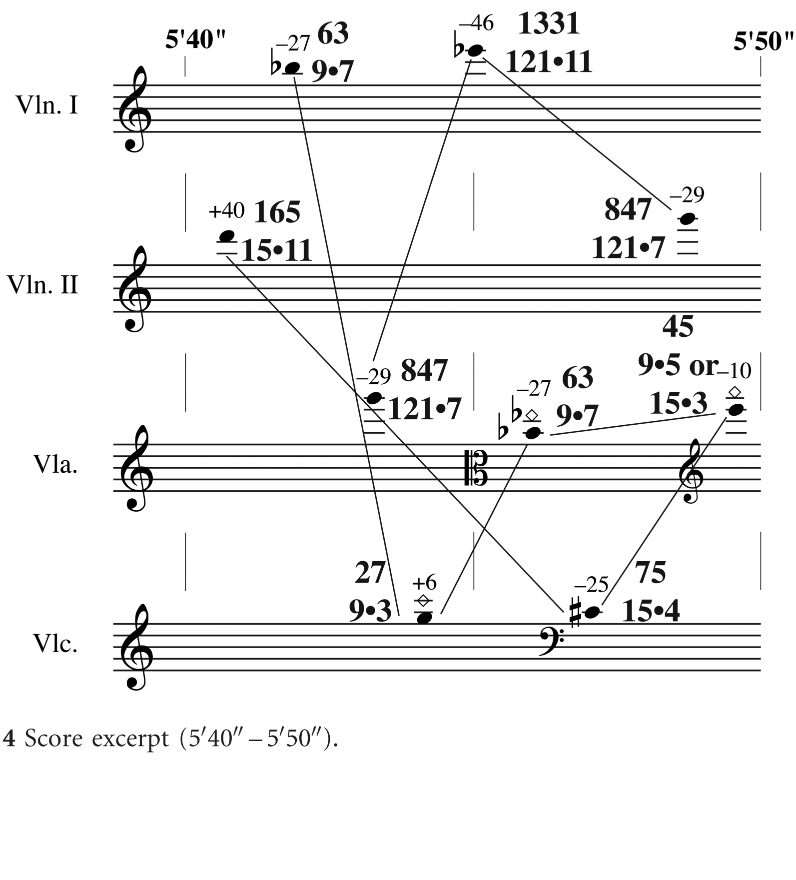

Figure 4 Score excerpt (5′40″ – 5′50″).

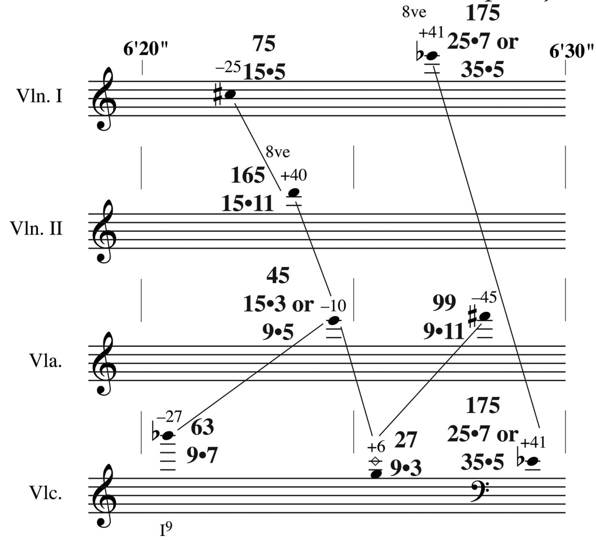

Figure 5 Score excerpt (6′20″ – 6′30″).

bottom numbers results in the top number. For example, the set of pitches comprising B-flat −27¢, E −10¢, G +6¢ and F-sharp −45¢ in Figure 5 can be heard and analyzed as implying one tonality since the harmonics (in relation to the fundamental) they are derived from (63, 45, 27 and 99, respectively) have a common divisor of 9 (C +4¢). Thus, they are in the tonality of C +4¢ and can be analyzed as the 7th, 5th, 3rd and 11th harmonics, respectively, of the 9th harmonic of B-flat.

Eventually, members of the entire pitch set sound including the B-flat, the fundamental of the entire piece. Figure 6 shows all the pitches in the piece, along with their rational ratios, absolute interval size and cents deviations from the nearest pitch in 12-tone equal temperament. As mentioned in the introduction, time-variant pitch ranges, amplitude contours and density changes also articulate the form of *Arbor Vitae*. These are discussed in the following section.

The Algorithm

Calculating Roots

Each possible root (*rt*) has two corresponding variables (*rtprob* and *rtpsum*) directly related to its probability.[6] The *rtprob* of each *rt* is initialized to $\frac{1}{\sqrt{rt}}$. *rtpsum* is the sum of the *rtprob*s of all roots on the same diagonal that are less than or equal to that particular root. For example, $rt = 33$ lies on the 3rd diagonal and the initial

1st Diagonal

Ratio	Cents	Pitch Class
1/1	0	B-flat +0¢

2nd Diagonal

Ratio	Cents	Pitch Class
3/2	702	F +2¢
5/4	386	D –14¢
7/4	969	A-flat –31¢
11/8	551	E –49¢

3rd Diagonal

Ratio	Cents	Pitch Class
9/8	204	C +4¢
15/8	1088	A –12¢
21/16	471	E-flat –29¢
25/16	773	F-sharp –27¢
33/32	53	B –47¢
35/32	155	C –45¢
49/32	738	F +38¢
55/32	938	G +38¢
77/64	320	C-sharp +20¢
121/64	1103	A +3¢

4th Diagonal

Ratio	Cents	Pitch Class
27/16	906	G +6¢
45/32	590	E –10¢
63/32	1173	B-flat –27¢
75/64	275	C-sharp –25¢
99/64	755	F-sharp –45¢
105/64	857	G –43¢
125/64	1159	B-flat –41¢
147/128	240	C +40¢
165/128	440	D +40¢
175/128	541	E-flat +41¢
231/128	1022	A-flat +22¢
245/128	1124	A +24¢
275/256	124	B +24¢
343/256	506	E-flat +6¢
363/256	605	E +5¢
385/256	706	F +6¢
539/512	89	B –11¢
605/512	289	C-sharp –11¢
847/512	871	G –29¢
1331/1024	454	E-flat –46¢

Figure 6 Entire pitch set of *Arbor Vitae*. The numerators of the 'Ratios' correspond to the harmonic structure (illustrated in Figure 1) and each denominator is set to the greatest power of 2 that is less than the numerator (moving all pitches to within one octave). The 'Cents' column gives the absolute interval size (from the 1/1). 'Pitch Class' is given with a cents deviation from the nearest tempered pitch. Pitch spellings in this figure and in the score are consistent with spellings in Tenney's notes.

$rtpsum = \frac{1}{\sqrt{33}} + \frac{1}{\sqrt{25}} + \frac{1}{\sqrt{21}} + \frac{1}{\sqrt{15}} + \frac{1}{\sqrt{9}} \approx 1.18383$. Then, the sum of all *rtprob*s on a given diagonal (*dpsum*) is calculated. *dpsum* is equal to the *rtpsum* of the largest root on that diagonal. The initial *dpsum* for the 3rd diagonal can be written as follows:

$$dpsum = \frac{1}{\sqrt{121}} + \frac{1}{\sqrt{77}} + \frac{1}{\sqrt{55}} + \frac{1}{\sqrt{49}} + \frac{1}{\sqrt{35}} + \frac{1}{\sqrt{33}} + \frac{1}{\sqrt{25}} + \frac{1}{\sqrt{21}} + \frac{1}{\sqrt{15}} + \frac{1}{\sqrt{9}} \approx 1.83543.$$

Starting from time 0, successive roots are chosen to calculate branches. Each chosen root (*crt*) has a corresponding start-time (*strt*).[7] The diagonal a chosen root lies on (*rdiag*) is determined by the following piecewise equation over time:

$$rdiag = \begin{cases} 3 & strt < 100 \\ 2 & strt < 260 \\ 1 & strt \geq 260 \end{cases}$$

To choose a root, a random number (*rand*) is generated such that $rand \in \mathbb{R}$ and $0 \leq rand \leq dpsum$. The chosen root (*crt*) is the *rt* on the current diagonal (*rdiag*) with the next greatest *rtpsum* in relation to *rand*. For example, at time 0 ($rdiag = 3$), if $rand = 0.84276$, then $crt = 25$, since the *rtpsum* of 25 is approximately 1.00975 and the *rtpsum* of the next lowest root, 21, is app 0.80975.

After every *crt* is determined, the *rtprob*s of all roots on the current *rdiag* (except for the root equal to *crt*) are recalculated as follows:

$$\text{if } (rtprob == 0), \text{then } rtprob = \frac{1}{\sqrt[4]{rt}};$$
$$\text{else if} \left(rtprob == \frac{1}{\sqrt[4]{rt}} \right), \text{then } rtprob = \frac{1}{\sqrt[3]{rt}};$$
$$\text{else if } \left(rtprob == \frac{1}{\sqrt[3]{rt}} \right), \text{then } rtprob = \frac{1}{\sqrt{rt}};$$

Then, *rtprob* of the root equal to *crt* is set to 0 and all the *rtpsum*s and the *dpsum* of that diagonal are recalculated. Thus, after a root is chosen, three more roots must be chosen until the *rtprob* of that root returns to its initialized value. Note that at the beginning of the piece and every time *rdiag* changes, roots more closely related to the fundamental are favored because of how the *rtprob*s are initialized. The recalculation of probabilities after a root is chosen ensures that the same root will not be chosen twice in a row and that there will be many different roots chosen throughout the piece. That is, there will be a quasi-uniform distribution of roots.

The duration of a chosen root (*rdur*) is the amount of time during which branches are calculated from that root. As the root durations decrease (roots are chosen more frequently), the harmonies shift more rapidly. *rdur* is determined by the variables *exrmax*[8] (the maximum root duration exponent) and *exrmin* (the minimum root duration exponent). *exrmax* is calculated by the following piecewise function over time:

$$exrmax = \begin{cases} 5 - .5\frac{strt}{160} & strt < 160 \\ 4.5 + .25\frac{strt-160}{100} & strt < 260 \\ 4.75 - .5\frac{strt-260}{80} & strt < 340 \\ 4.25 + .25\frac{strt-340}{80} & strt < 420 \\ 4.5 - .5\frac{strt-420}{60} & strt < 480 \\ 4 & strt < 600 \\ 4 + \frac{strt-600}{180} & strt \geq 600 \end{cases}$$

exrmin is always $exrmax - 1$. A random number (*rand*) is generated such that $rand \in \mathbb{R}$ and $exrmin \leq rand \leq exrmax$ then $rdur = 2^{rand}$. The following graphs show the range between *exrmax* and *exrmin* (Figure 7) and the range of *rdur* (Figure 8) throughout the piece.

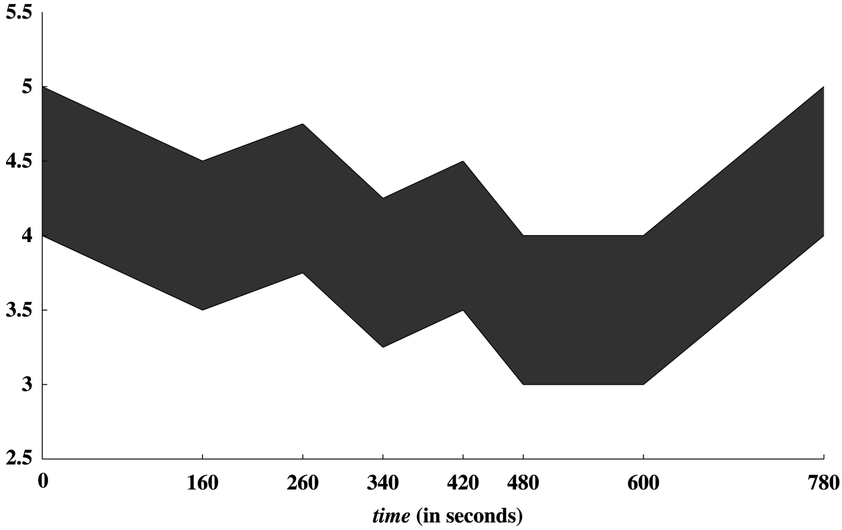

Figure 7 Range between *exrmax* and *exrmin* over time.

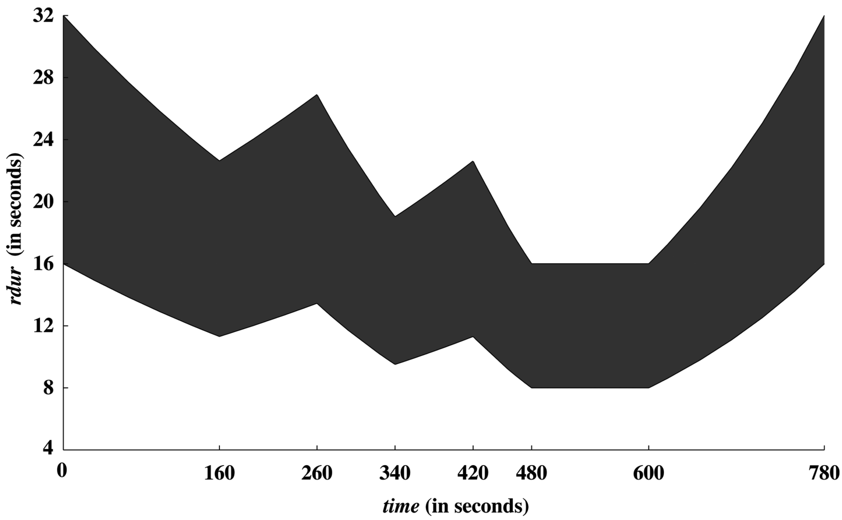

Figure 8 Range of *rdur* over time.

Calculating Branches

For every chosen root, from the start time of the root (*strt*) to $strt + rdur$, several branches are calculated using the variables *nmult* and *canReqB*. *nmult* is the number of multiplications that will be performed on the currently chosen root to calculate a branch. *canReqB* is an integer indicating whether or not a branch can equal the chosen root: 0 if it can and 1 if it cannot. *nmult* and *canReqB* are determined by the following piecewise equations over time:

$$nmult = \begin{cases} 1 & strt < 100 \\ 2 & strt < 260 \\ 3 & strt \geq 260 \end{cases}; \quad canReqB = \begin{cases} 1 & strt < 60 \\ 0 & strt < 100 \\ 1 & strt < 160 \\ 0 & strt < 260 \\ 1 & strt < 420 \\ 0 & strt \geq 420 \end{cases}$$

Together with *rdiag*, these variables determine the set of diagonals on which a branch can lie (*bdiagset*). *bdiagset* is constructed of all diagonals from $rdiag + canReqB$ to $rdiag + nmult$. In set notation:

$$bdiagset = \{rdiag + canReqB, rdiag + canReqB + 1, \ldots, rdiag + nmult\}$$

- Example 1: If *strt* is at 10 seconds, branches will only lie on diagonal 4 since $rdiag = 3$, $nmult = 1$ and $canReqB = 1$.
- Example 2: If *strt* is 250 seconds, branches can lie on diagonals 2, 3 and 4 since $rdiag = 2$, $nmult = 2$ and $canReqB = 0$.

Together, *rdiag* and *bdiagset* delineate six differences between the constructions of harmonies over the course of the piece. Figure 9 shows when these different 'states' occur, which are expressed by the following piecewise equation:

$$6 \text{ changes} = \begin{cases} 1)rdiag = 3; & bdiagset = \{4\} & strt < 60 \\ 2)rdiag = 3; & bdiagset = \{3, 4\} & strt < 100 \\ 3)rdiag = 2; & bdiagset = \{3, 4\} & strt < 160 \\ 4)rdiag = 2; & bdiagset = \{2, 3, 4\} & strt < 260 \\ 5)rdiag = 1; & bdiagset = \{2, 3, 4\} & strt < 420 \\ 6)rdiag = 1; & bdiagset = \{1, 2, 3, 4\} & strt \geq 420 \end{cases}$$

Note that *bdiagset* is the same from sections 2 to 3 and 4 to 5. However, the fact that *rdiag* changes between these sections affects how branches are chosen. This will become clear after the explanation of a branch calculation.

Each chosen branch has a corresponding start-time (*stbr*). Starting from time 0, successive branches are calculated by *nmult* multiplications on *crt*. Each multiplication is by 1, 3, 5, 7 or 11. Like *rt*, each one of these multipliers (*mult*) has two corresponding variables (*multprob* and *multpsum*) directly related to its probability. Every time a new root is chosen, the *multprob* for each multiplier is set to

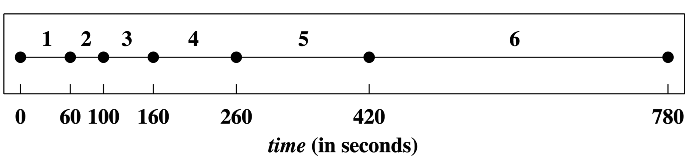

Figure 9 Six harmonic states over time.

$\frac{1}{\sqrt{mult}}$ and *multpsum* is the sum of the *multprob*s of all multipliers less than or equal to that particular multiplier. For example, directly after a new root is chosen, $mult = 7$ has a $multpsum = \frac{1}{\sqrt{7}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{3}} + 1 \approx 2.40253$. Then, the sum of all *multprob*s (*msetpsum*) is calculated. *msetpsum* is equal to the *multpsum* of the largest multiplier, 11. For the first multiplication, a random number (*rand*) is generated such that $rand \in \mathbb{R}$ and $canReqB < rand \leq msetpsum$. Thus, a branch cannot equal the chosen root if $canReqB = 1$. For each successive multiplication, $rand \in \mathbb{R}$ and $0 < rand \leq m$-*setpsum* (allowing a branch to equal the chosen root). The chosen multiplier (*cmult*) equals the *mult* with the next greatest *multpsum* in relation to *rand*. For example, within the first minute of the piece and directly after a new root is chosen ($rdiag = 3$, $nmult = 1$, and $canReqB = 1$), if $rand = 2.25867$, then $mult = 7$, since the *multpsum* of 7 is approximately 2.40253 and the *multpsum* of the next lowest multiplier, 5, is approximately 2.02456. If *crt* is 15, then $br = 15 \cdot 7 = 105$.

The calculation of a branch can be written as follows:

$$
\begin{aligned}
&br = crt;\\
&\text{for (int } i = 0; i < nmult; i++)\{\\
&\text{choose } cmult;\\
&br = br \cdot cmult;\\
&\}
\end{aligned}
$$

Once a branch is chosen, multiplier probabilities are recalculated as follows:

$$
\begin{aligned}
&\text{if } (multprob == 0), \text{then } multprob = \frac{1}{\sqrt[4]{mult}};\\
&\text{else if}\left(multprob == \frac{1}{\sqrt[4]{mult}}\right), \text{then } multprob = \frac{1}{\sqrt{mult}};
\end{aligned}
$$

Then, the *multprob* of the multiplier equal to *cmult* is set to 0 and all the *multpsum*s and the *msetpsum* are recalculated. Note that these values are not recalculated upon every multiplication; they are only recalculated *after a branch is chosen* (after *nmult* multiplications). Otherwise, certain branches could not be chosen. For example, if the *multprob* of 3 is set to 0 after being chosen on the first multiplication and *nmult* is 2, then a branch cannot equal $rt \cdot 3 \cdot 3$.

Directly after a root is chosen, lower multipliers have a higher probability because of how the *multprob*s are reset. However, as with choosing roots by their corresponding *rtprob*s, the recalculation of the *multprob*s ensures a quasi-uniform distribution of the multipliers over time. Note that branches more closely related to the chosen root do not necessarily have a higher probability of occurring. As *nmult* increases, branches that are compound integers (i.e., have more divisors) may occur more often.[9]

The duration of a branch (*bdur*) is directly related to *rdur* because the maximum branch duration exponent (*exbmax*) is always equal to $exrmax - 2.5$. Shorter branch durations result in a higher temporal density and, as with shorter root durations, more rapidly changing harmonies.

Even though several branches may be calculated upon a given root, *exbmax* is only recalculated when a new root is chosen since it is based on *exrmax*. *exbmax* is not recalculated or interpolated for every new branch. The minimum branch duration exponent (*exbmin*) is always $exbmax - 1$. A random number is generated such that $rand \in \mathbb{R}$ and $exbmin \leq rand \leq exbmax$ then $bdur = \frac{2^{rand}}{2}$.[10] The following graphs show the range between *exbmax* and *exbmin* (Figure 10) and the range of *bdur* (Figure 11) throughout the piece.

Deriving Pitches from Branches

For every branch, there is a series of calculations to derive a pitch that is within the available pitch range at that moment. The available pitch range over time (graphed in Figure 12) has *low* and *high* limits (in cents from the fundamental) calculated by the following piecewise equations:[11]

$$low = \begin{cases} 7800 - 1200\frac{stbr}{40} & stbr < 40 \\ 6600 & stbr < 100 \\ 6600 - 1800\frac{stbr-100}{60} & stbr < 160 \\ 4800 & stbr < 260 \\ 4800 - 2400\frac{stbr-260}{80} & stbr < 340 \\ 2400 & stbr < 420 \\ 2400 - 1800\frac{stbr-420}{60} & stbr < 480 \\ 600 & stbr < 600 \\ 600 + 5400\frac{stbr-600}{180} & stbr \leq 780 \end{cases}; \quad high = \begin{cases} 7800 & stbr < 420 \\ 7800 - 5400\frac{stbr-420}{180} & stbr < 600 \\ 2400 + 5400\frac{stbr-600}{180} & stbr \leq 780 \end{cases}$$

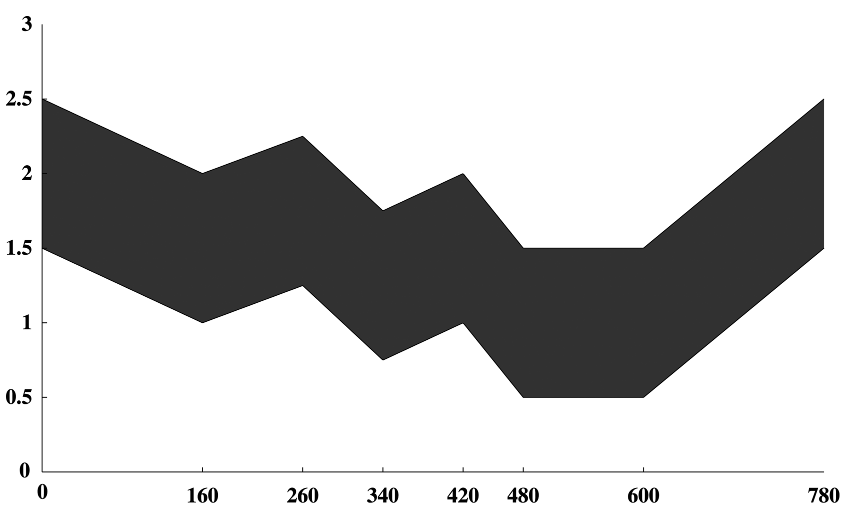

Figure 10 Range between *exbmax* and *exbmin* over time.

For each branch, the *low* and *high* values are calculated based on *stbr* and a pitch (*lpc*) (in cents from the fundamental) is derived from the branch such that $lpc = \log_2(br) \mathrm{mod} 1200$. Note that this pitch is within one octave of the fundamental,

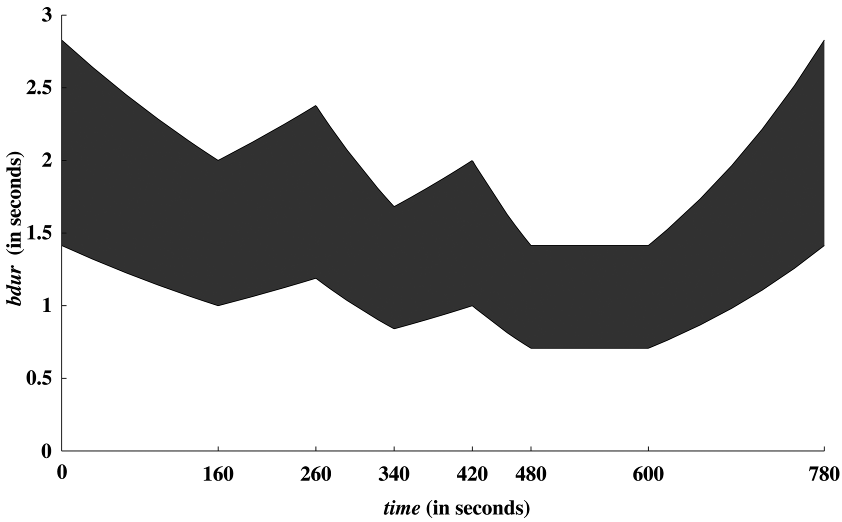

Figure 11 Range of *bdur* over time.

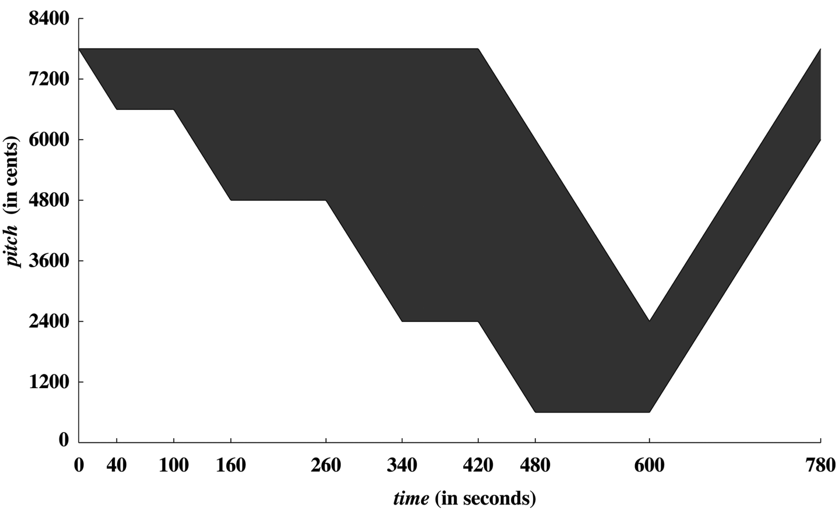

Figure 12 Pitch range over time.

B-flat$_1$. Then, a set of integer multipliers (*imultset*) is determined such that, when multiplied by 1200 and added to *lpc*, is a pitch between the *low* and *high* values.[12] In set notation:

$$imultset = \{m | m \in \mathbb{Z} \text{ and } low \leq lpc + 1200m \leq high\}.$$

The sounding pitch is determined by choosing a number from *imultset* at random, multiplying it by 1200, and then adding that to *lpc*. For example, if $br = 15$ and $stbr = 420$, then $lpc = 1088$, $low = 2400$, $high = 7800$ and $imultset = \{2, 3, 4, 5\}$. If 2 is chosen randomly from *imultset*, then the pitch is placed 3488¢ (which equals $1088 + 1200 \cdot 2$) above B-flat$_1$ (the sounding pitch is A$_4$ -12¢).

Assigning Tones to Instruments and Temporal Density

The pitch and the corresponding start-time of a tone (*stbr*) is assigned to a particular instrument based on the following guidelines and exceptions. First, after a tone is assigned to an instrument, that instrument cannot be assigned a new tone until at least two tones have been assigned to other instruments. This is to ensure that tones are distributed more or less evenly among the instruments. Second, Rule 1 may only be broken when the pitches of two or more successive tones fall below the violins' low F. (String IV of each violin is tuned down; see the section *String Tunings and Timbre*, below). In this case, the tones are assigned to the viola and cello in alternation. And third, Rules 1 and 2 may only be broken when the pitches of two or more successive tones fall below the viola's B-flat. (String IV of the viola is also tuned down.) In this case, the cello can receive several tones in a row.

Since the tones are more or less evenly distributed to the different instruments throughout much of the piece, the average tone duration is usually approximately four times greater than the average branch duration. The exceptions (2 and 3) primarily occur after 420 seconds when the upper limit of the available pitch range descends. As a result, the viola and cello play shorter, lower tones than the violins play. Note that the only time a tone's duration is equal to *bdur* is when the cello is assigned more than one tone in a row. Since a pitch's duration is usually longer than the duration of the branch deriving the pitch, the harmonic transitions typically overlap.

The Flow of the Algorithm

After an instrument is assigned a tone, the start-time of the branch is incremented by the branch duration and a new branch is calculated. Once *stbr* is greater than the start-time of the current root plus the root duration, the start-time of the root is incremented by the root duration and a new root is chosen. The entire length of the work is 780 seconds (13 minutes). The general algorithm of the piece can be written in pseudo-code as follows:

```
strt = 0;
stbr = 0;
initialize rtprobs, rtpsums, and dpsum;
while(strt < 780){
   choose rt;
   recalculate rtprobs, rtpsums, and dpsum
   calculate rdur;
   (re)set multprobs, multpsums, and msetpsum;
   while(stbr < strt + rdur){
      calculate br;
      recalculate multprobs, multpsums, and msetpsum;
      calculate bdur;
      calculate pitch placement;
      assign tone to an instrument;
      stbr = stbr + bdur;
   }
   strt = strt + rdur;
}
```

String Tunings and Timbre

Subtle, gradual changes in timbre occur throughout the piece based on the time-variant pitch range and the tunings of the strings (Figure 13), which are designed to enable as many tones as possible to sound as natural harmonics. All the open strings are octave equivalents of harmonics 1, 3, 5, 7 and 11 of B-flat. As the lower limit of the pitch range descends, more and more pitches are played as natural harmonics.

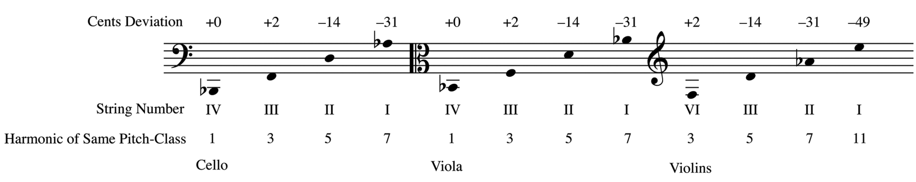

Figure 13 String tunings.

Dynamics

The loudness contour of the piece comprises three sections: a crescendo, a loud plateau and then a decrescendo. In the score, the dynamics go from *pianissimo* to *forte* and back, with intermediary levels linearly interpolated as shown in Figure 14. The dynamics refer to the entire ensemble.

After the Algorithm: Score Generation

The algorithm was implemented in a computer application written in the BASIC programming language. The application generated a list of pitches, each with a corresponding start-time and instrumental assignment. The list was formatted for an automatic transcriber with an accompanying playback module that I developed for Tenney. The transcriber was written in Java using JMSL and JScore (Didkovsky, 1997 – 2007), which have versatile transcription objects. Tenney made decisions based on these transcriptions, synthesized playbacks, data printouts and his own graphs. After running the BASIC program and checking the code and output data several times, Tenney felt that the algorithm was consistent and that all realizations would be essentially equivalent. Nonetheless, he still wanted to choose from three generated versions. After listening to synthesized playbacks of all three versions and studying the output data, he selected the second realization. The current score is a transcription of that data.

Additionally, Tenney was very specific about the score format (2 systems per page, 4 measures per system, 5 seconds per measure, with a proportional ‘space equals time’ notation). He also decided to notate all pitches more than two octaves above the highest open string of each instrument as artificial harmonics, along with a performance instruction allowing the option of playing these tones by stopping the

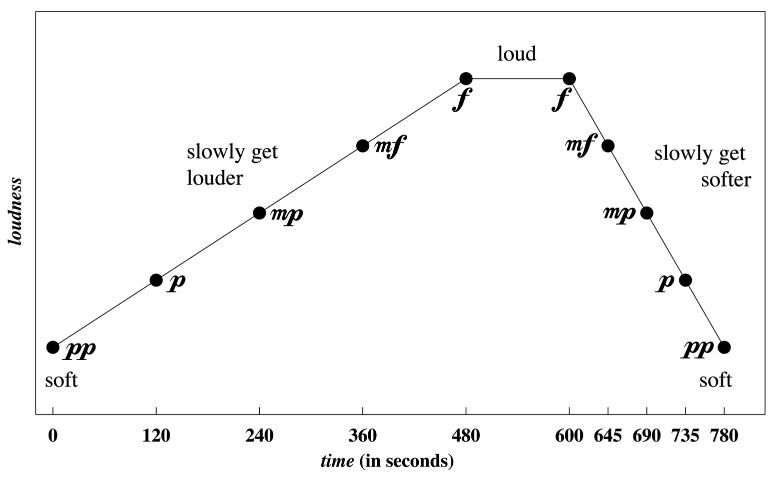

Figure 14 Loudness contour.

string if more comfortable for the performer. Many of these notational conventions emanate from previous pieces. Tenney frequently referred to these pieces and also reviewed old computer code. In particular, the distribution of tones among parts is similar to the distribution of tones among voices in the piano piece, *To Weave (a meditation)* (2003), and certain notational conventions, such as those for natural harmonics, come from *Diapason* (1996).

Tenney passed away before proofreading a final score. A small group of people familiar with his work helped make the final edition: Michael Pisaro, Michael Byron and Larry Polansky to name a few. A high priority was given to making the score of *Arbor Vitae* consistent with Tenney's previous notational conventions, which were used as precedents. The scores for *Diapason* (1996), *To Weave (a meditation)* (2003) and the *Spectrum* series (1995–2001) were particularly important. *Arbor Vitae's* performance instructions were also modeled on the instructions from these scores.

History of *Arbor Vitae*

Jim and the Bozzini Quartet first discussed the creation of a new piece in November 2004 at a festival for Jim's 70th birthday in Los Angeles. Jim began working on what would become *Arbor Vitae* within the next several months. Though there is no 'official' commission date, the Bozzini Quartet applied to the Canada Council for the Arts to assist the commission and received a positive reply in March 2006. They premiered *Arbor Vitae* on 10 December 2006 at the California Institute of the Arts during a festival celebrating the life and work of James Tenney.

In the spring of 2006, while living in Vienna, I was informed of James Tenney's diagnosis of a progressed cancer. Upon returning to the United States in late June, I flew straight to Los Angeles to see him. During this first visit after the diagnosis, Jim seemed well and optimistic despite the side effects of the chemotherapy. We spoke about the projects he wanted to finish. These projects included publishing a book of articles, finalizing a multiple-pitch perception algorithm and finishing his string quartet, *Arbor Vitae*.

During my stay, I perused Jim's notes and sketches for *Arbor Vitae* and began to understand how the piece worked. On the day prior to my departure, Jim explained it to me. The piece was completely planned out: conceptually finished. The main problem, given Jim's illness, was writing the computer program to generate the piece. On the last day of my visit, we developed a style of working together. As he talked me through the steps of the algorithm, I made notes then programmed the steps, modifying the BASIC program he had already begun. We made excellent progress that day. This revitalized his hopes of finishing *Arbor Vitae* and moving on to the other projects. Before leaving, I explained to Jim the changes and additions I made to his code.

Later in the summer of 2006, Lauren Pratt, Jim's wife, asked if I would return to Los Angeles to help Jim finish the piece. I went back to Los Angeles a week after Lauren's request. Immediately on arriving, Jim and I got to work and made rapid progress. Jim was enthusiastic about our working dynamic. For a week we worked intensely, often more than ten hours a day. Jim needed to take rest breaks, which allowed me to keep pace. On the evening of the day in which we first generated the piece from beginning to end, Jim was ecstatic. The next day, Jim's health deteriorated somewhat and our progress slowed a bit. Nevertheless, the piece was finished before I left. Less that two weeks later, after finishing what would be his last piece, James Tenney passed away.

Acknowledgements

It was an honor to work with Jim Tenney during his completion of *Arbor Vitae*. While working, Jim, with his characteristic clarity and elegance, made all aspects of this difficult piece understandable. Jim was a vital and nurturing part of the 'tree of life'. He championed the works of many composers, both predecessors and contemporaries. His contribution to the modern musical repertoire is prodigious. Most personally, he taught and inspired countless younger composers. It is these generous contributions for which I and I know many others thank him deeply.

A special thanks to Lauren Pratt and James Tenney's family, Larry Polansky, Michael Pisaro, Michael Byron, Nick Didkovsky and the Bozzini Quartet, all of whom contributed to the current edition of the score. And thanks to Lauren Pratt, Mark So, Ted Coffey and especially Larry Polansky, all of whom helped with this article.

Notes

[1] A subscript attached to a note name indicates octave placement. B-flat$_1$ denotes 2 octaves and a major second below middle C, which would be denoted as C_4.

[2] The pitches of *Arbor Vitae* are in an 11-limit just-intonation (see Partch (1974) on intonation limits).

[3] Deviation from the nearest pitch in 12-tone equal temperament is expressed in cents (one hundredth of a tempered semitone) using a minus or plus sign and the cents symbol, ¢.

[4] All score examples and manuscripts of *Arbor Vitae* reprinted by permission.

[5] Note that the 15th partial is approximately 11.73¢ flat from the nearest equal tempered pitch, and the 5th partial is approximately 13.69¢ flat. In *Arbor Vitae*, cents deviations are only rounded from the harmonic of B-flat from which the pitch is derived.

[6] Throughout the article, variables are named similarly to the actual names of variables that Tenney used.

[7] All time variables are in seconds.

[8] Before I worked with Tenney on *Arbor Vitae*, he had already defined an algorithm to generate the piece. However, during the time we worked together, Tenney made a few changes to his original algorithm. A change in *exrmax* was one of these. Originally, the variable spanned from 4 to 6 instead of 4 to 5. This change was made so that roots were chosen more frequently throughout much of the piece.

[9] A close examination of the time-variant probability schemes implemented in *Arbor Vitae* shows that the possibility of a given branch depends on *rdiag*, *nmult*, *canReqB*, the *rtprob*s and the *multprob*s, which are ultimately related to the size and number of prime factors of that branch – that is, a branch's probability is based on the harmonic distance with respect to

the given root and the fundamental. Also, the probabilities are continuously being recalculated so that the possibility of roots and branches that have not been chosen for an extended period of time increases. An in-depth analysis of this complex, time-variant probabilistic system and its musical result exceeds the scope of this article.

[10] The division by 2 for *bdur* shortens the average sounding tone durations throughout the piece. It was another one of the few changes Tenney made from the original algorithm in which $bdur = 2^{rand}$.

[11] These piecewise equations were derived from a graph that I transcribed with Tenney after he decided to alter his original one. In Tenney's original graph, which uses a linear pitch scale, some of the breakpoints determining the limits of the pitch range are drawn midway between the B-flats at the vertical position of a tritone, but the vertical axis is labeled with F +2¢ at those vertical positions suggesting that the octaves be split harmonically into a just fifth and a just fourth: breakpoints at B-flat and F +2¢. In the transcribed graph, these breakpoints are drawn at the same vertical position as in the original graph, but are labeled according to their vertical position on a linear pitch scale thus splitting the octave equally into two tritones: breakpoints at B-flat and E. Neither Tenney nor I noticed the discrepancy between the two versions while studying the data outputs or listening to synthesized realizations of the piece. The change affects only two pitch classes in the entire set of available pitch classes of the piece: the E +5¢ (derived from the 363rd partial of B-flat) and the F +2¢ (derived from the 3rd partial of B-flat) may have been assigned to different registers.

[12] For the first 40 seconds, *imultset* may be empty since the pitch range is less than an octave. In this case, the branch is discarded. This accounts for a silence at the beginning of the piece.

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