Commas

for Ed Carroll

Michael Winter (2005)

Commas *for Ed Carroll*

When a prime number is raised to any power greater than zero it will never equal another prime number raised by any power; however, it may come very close. These near misses are called commas and are the phenomena explored in this work. The fact that commas exist is one of the fundamental issues of tuning theory. One well-documented comma is the Pythagorean comma, which is the interval created when twelve stacked perfect fifths come close to seven stacked octaves. In theory, there are an infinite number of commas.

Notation

The score is composed of two textures labeled Texture I and Texture II. Texture II is a set of available pitches that are played throughout the piece and Texture I is a sequence of segments with pitches that are determined by primes raised to powers.

Above each note is a deviation in cents, one-hundredth of a semitone, from the closest written pitch in a twelve-tone equal-tempered tuning system. Below each note is a harmonic number, which represents the position of that pitch in the harmonic series if it was not reduced into the octave of the reference fundamental, represented by the under-number 1. Even though the pitches are displayed in one octave they may be transposed by octaves in either direction, e.g. the written pitch represents a pitch class that should be played within the pitch range determined by the lower graph in the score (See ***Pitch Range***).

The entire work may be transposed by using a different pitch as a reference fundamental. A blank score is included for players to write in such transpositions.

At any point past five segments, which are marked by measures in Texture I, the ensemble can retrograde or play the optional middle section and then retrograde. The arrows that go back and forth in the score document this.

Included with the score, is an addendum that documents the realization of the first performance of this work.

Texture I

Each segment, represented by a measure, contains available pitches based on stacked intervals, which are then reduced and displayed in one octave. Each successive segment overlaps the previous segment by fifteen seconds. Players should attempt to play pitches that are close to pitches that are sounding from the Texture II players.

The under-numbers in Texture I are the result of the expression

$$\text{Prime}^{x}\}_{x\geq 0}^{7}.$$

In the score, each segment displays x from zero to the seventh degree, but the available pitch set could include pitches that extend the values of x to a much greater degree and can theoretically extend infinitely;

$$\text{Prime}^{x}\}_{x\geq 0}^{\infty}.$$

To determine the frequency ratio of a particular octave-reduced interval in relation to a fundamental divide the result of the previous equation by the greatest power of 2 that is less then the numerator, e.g.

$$frequency\ ratio = \frac{\text{Prime}^{x}}{2^{z}}\Bigg\}$$ so that 2^{z} is as great as possible but less than Prime^{x}.

One can calculate the nearest pitch in a twelve-tone system by determining the interval in cents by solving this equation;

$$Interval\ in\ cents = \frac{1200}{\log_{10} 2}(\log_{10} frequency\ ratio).$$

Since a semitone is one hundred cents, one can subtract the interval in cents from the closest interval in a twelve-tone system to determine the cent deviation from the written note.

A more extensive list of the possible pitches for each segment is given on the second page of the score.

Once a segment begins, all written pitches in musical notation and the pitches that can be determined by the process above can be played till the end of that segment in any order. The linear documentation on the page should not necessarily be interpreted to suggest that those pitches be played in succession.

Texture II

The pitches written in Texture II are played throughout the entire piece. They are the primes raised to the first power. The interval size and cent deviations can be calculated by the procedure listed above (see ***Texture I***). The number of segments the ensemble chooses to play (see ***Realization***) before the retrograde or optional middle section is related to the number of pitches (e.g. primes), which is related to the necessary number of performers/voices. The number of segments is always one less than the number of primes, and the number of performers/voices should be less than or equal to the number of primes and greater than or equal to the number of primes divided by two. (This is documented in the score under Texture II.) Players should try to play pitches that are not already sounding.

Ranges of Tone Durations and Dynamics

In the middle of the score is a graph that illustrates the possibilities of tone durations and dynamics for a performance of ten segments prior to the optional middle segment and/or retrograde. The boundaries off the gray area show the limits of possible tone durations and dynamics. For tone durations, the y-axis is linear from 0 to 3 seconds, but for dynamics it is inverted. Of utmost importance is that the middle of each successive segment should get louder with shorter tone duration creating a progression of increasing density and higher subjective intensity. During the overlapping portions of the segments the tone durations and dynamics should always retreat to between 1 and 3 seconds and between *pianissimo* and *mezzo piano*, respectively.

If the piece is played with more or fewer segments than ten, the lower limit can be recalculated by the equation to the left of the graphic. In essence, it makes it so that tones progress to as fast and as loud as possible by the final segment before the retrograde or performance of the optional middle section.

Pitch Range

At the bottom of the score is a graphic representation of the pitch ranges (illustrated by the gray area) that the ensemble should play within. The piece should start in the middle of the ensemble's range with a span of approximately one octave. By the last segment before the retrograde or optional middle section the ensemble should be playing throughout the ensemble's entire range.

Realization and Electronics

Prior to the performance, the ensemble should determine how many segments to play. Five is the minimum, but the ensemble is encouraged to play as many segments as possible to explore commas resulting from larger primes raised to powers. For example, the ensemble may choose to go to the 20^{th} prime, which is 71. Then there would be nineteen segments since 2^x is omitted. Texture II performers should then play all primes to 71 throughout.

There is a practice application that is either included or can be obtained so that the performers may familiarize themselves with the pitch vocabulary required for the work.

One may also use a computer application for performances with small ensembles or solo performers. The application should adhere to the following guidelines:

There should be separate signal processing of Texture I and II. The processing is granular synthesis with Gaussian envelopes imposed on the grains.

For Texture I, the user can choose to add one or two voices generated by grains. The program records the input signal into a buffer of ten seconds that continually rewrites after the initial ten seconds has been filled. Then, the program outputs one or two grains selected randomly from anywhere in the buffer at durations that satisfy the equation written to the left of the graphic representation of tone durations in the score but with the same dynamic as they were played.

The processing of Texture II works similarly to the processing of Texture I except that the number of grains is only limited by processing power and the length of the performance determines the buffer size. The buffer does not rewrite the contents. It continually records throughout the performance and the grains are extracted from any prerecorded part of the buffer. The number of primes (which is the number of segments plus one) minus the number of available performers determines the number of grains for Texture II. There is also a second option for a performance by one player. When this option is utilized, the performer can play all tones in Texture II into a buffer of indeterminate length. After the player records all the tones into the buffer and stops the record function, the application will granulate the signal according to the number of

segments previously determined and will satisfy the equation for tone durations, as in the processing for Texture I, and dynamics.

It is possible to write such an application in many programming environments. In essence, the program generates all the voices (with grains) not present in the ensemble so that the piece can be played by any number of performers. It is also acceptable to write a computer program that generates the textures stochastically so that all the parameters documented in the notes and in the score are satisfied. As these tools are created, they will be made available.

One Performer

The performer should start the record function of the application for Texture II. The performer then plays all the Texture II tones in all registers with 4-second durations before beginning the segments in Texture I. These tones should be played at an equal, strong dynamic level so that a hot signal from a microphone can be sent to the computer for processing. Once recording has begun, the player should try not to pause except for necessary breaths. When finished playing all the tones in Texture II, the player triggers the application to stop recording and can immediately start playing Texture I. If a computer application is used, the processing for Texture I should automatically begin once the record function for Texture II is stopped.

Two or More Performers

If using an application, trigger the application to start and begin playing Texture II till the texture is relatively saturated. Then Texture I may begin.

Since the pitch vocabulary in Texture I is large, the ensemble should divide the segments among the performers if the number of performers is greater than two. When the performers are not responsible for a segment in Texture I, they should return to playing the tones available in Texture II. This will create soloists or groups of soloists for each segment and will help differentiate the segments. If it takes more than one instruments to cover the indicated range at a particular segment, then those instruments are considered to comprise one voice.

Since the inputs to the computer processing will probably be different for Texture I and II, a larger ensemble that still utilizes computer processing should create a scheme that will allow the performers to play into the microphones that supply the signal into the Texture I or the Texture II processing of the application when necessary.

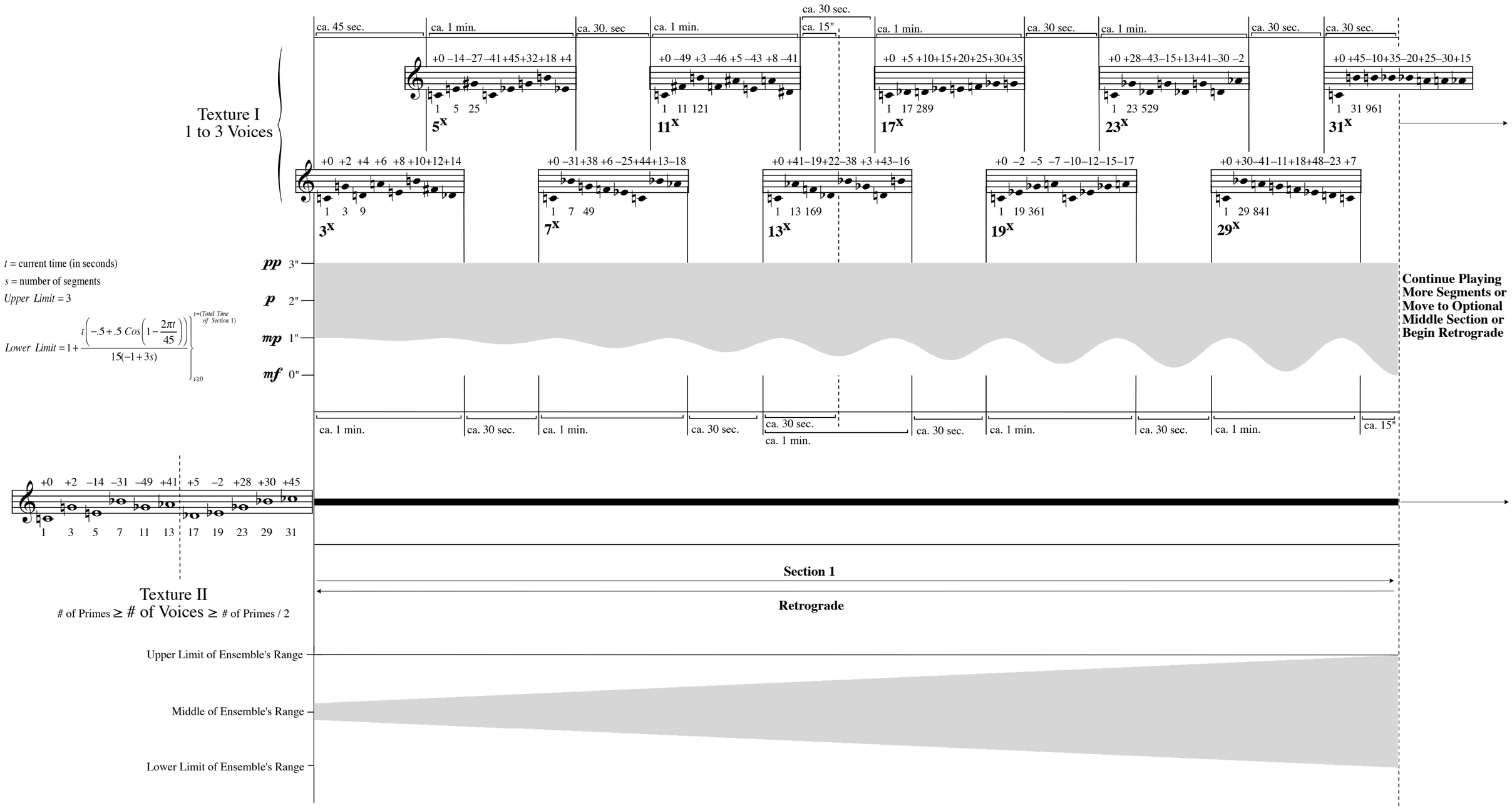
The Optional Middle Segment and the Retrograde

After performing a predetermined number of segments the performers may choose to play a middle section that consists of segments composed of any combination of the previously performed segments. The temporal format is essentially the same where segments overlap with each other by fifteen seconds. For example, the ensemble can perform the segments 5^x and 13^x together, and then play all segments at the same time. If the ensemble chooses to play the middle section, from at least three combinations to as many as possible should be performed before beginning the retrograde.

Commas

for Ed Carroll

Mike Winter (2005)

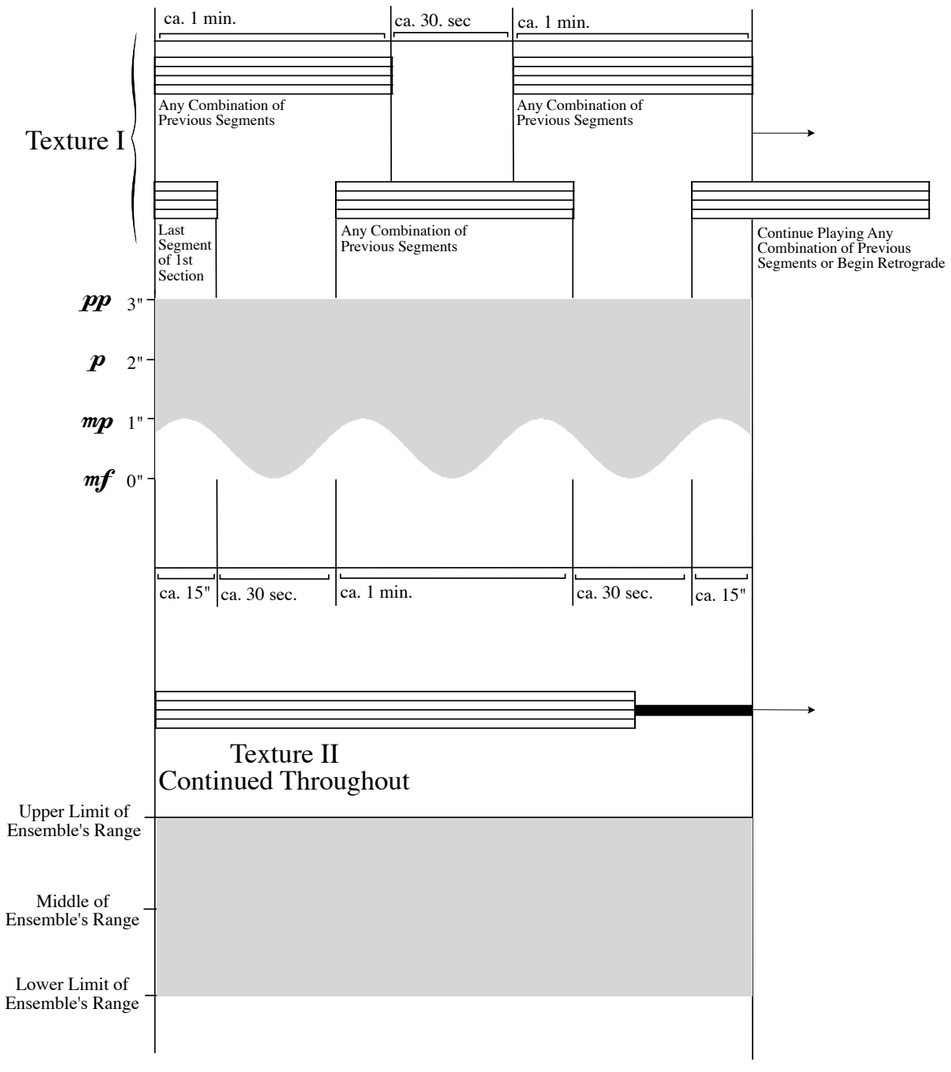


Commas

for Ed Carroll

Mike Winter (2005)

Optional Middle Section



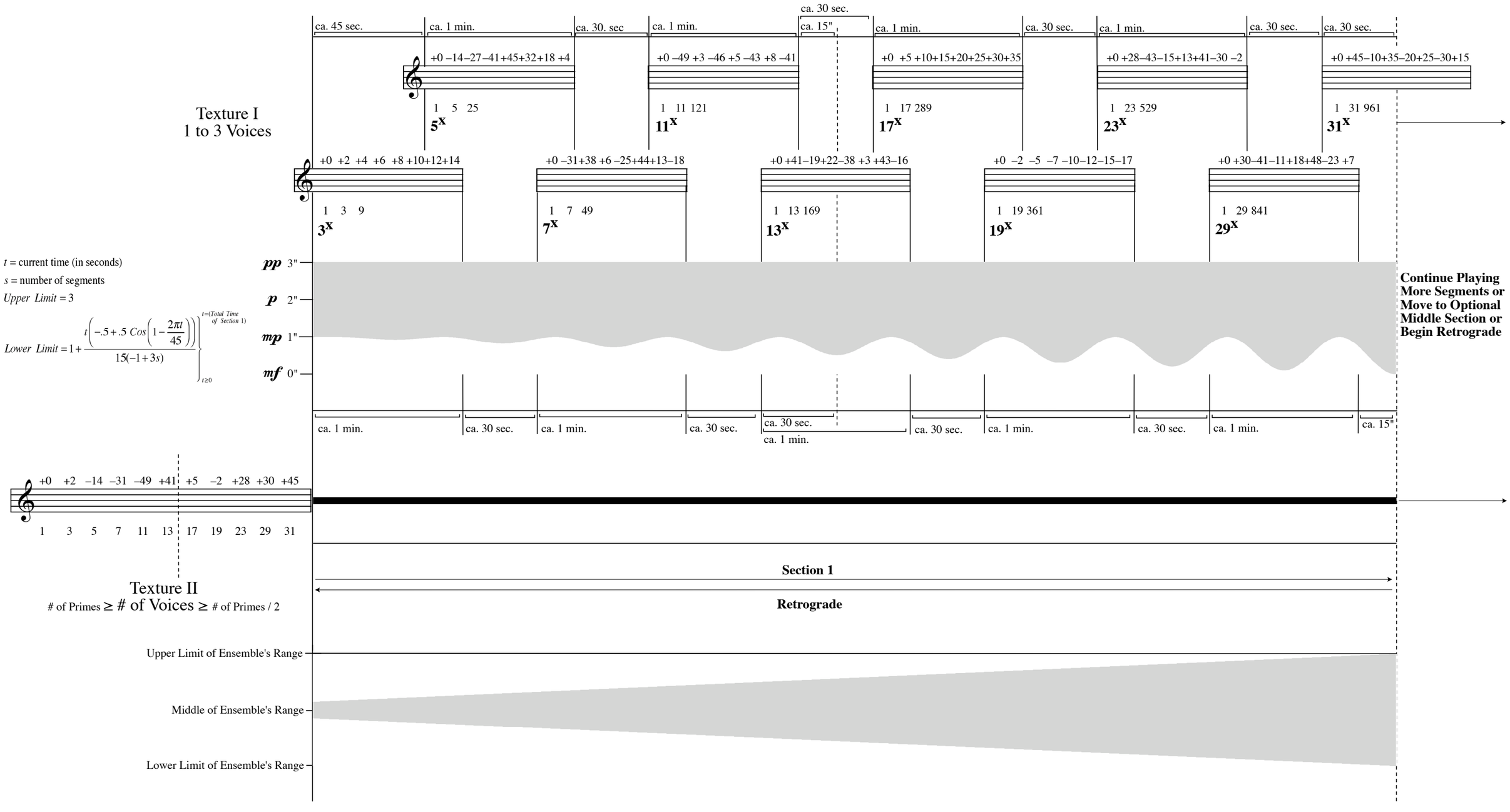
More Extensive List of Possible Pitches for Each Segment

x		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
3^x	Interval	P5	M2	M6	M3	M7	TT	m2	m6	m3	m7	P4	Uni	P5	M2	M6	M3	M7	TT	m2	m6	m3	m7	P4	Uni	P5	m3	m7	P4	Uni	P5
	Cent Dev.	2	4	6	8	10	12	14	16	18	20	22	23	25	27	29	31	33	35	37	39	41	43	45	47	49	-49	-47	-45	-43	-41
5^x	Interval	M3	m6	Uni	m3	P5	M7	m3	P5	M7	m3	TT	m7	M2	TT	m7	M2	TT	m7	m2	P4	M6	m2	P4	M6	m2	M3	m6	Uni	M3	m6
	Cent Dev.	-14	-27	-41	45	32	18	4	-9	-23	-37	49	36	22	8	-5	-19	-33	-46	40	26	13	-1	-15	-28	-42	44	30	17	3	-11
7^x	Interval	m7	P5	P4	m3	Uni	m7	m6	TT	m3	m2	M7	m6	TT	M3	m2	M7	M6	TT	M3	M2	M7	M6	P5	P4	M2	Uni	m7	P5	P4	m3
	Cent Dev.	-31	38	6	-25	44	13	-18	-49	19	-12	-43	26	-5	-36	32	1	-30	39	8	-23	45	14	-17	-48	21	-11	-42	27	-4	-35
11^x	Interval	TT	M7	P4	m7	M3	M6	m3	m6	M2	P5	m2	TT	Uni	P4	M7	M3	m7	m3	M6	M2	m6	m2	P5	Uni	TT	M7	P4	m7	M3	M6
	Cent Dev.	-49	3	-46	5	-43	8	-41	11	-38	13	-36	16	-33	18	-30	21	-28	24	-25	26	-22	29	-20	32	-17	34	-14	37	-12	40
13^x	Interval	m6	P4	m2	m7	TT	M2	M7	P5	M3	Uni	m6	P4	m2	m7	TT	M2	M7	P5	M3	Uni	M6	P4	m2	m7	TT	m3	M7	P5	M3	Uni
	Cent Dev.	41	-19	22	-38	3	43	-16	24	-35	5	46	-14	27	-33	8	48	-11	29	-30	11	-49	-8	32	-27	13	-46	-6	35	-25	16
17^x	Interval	m2	M2	m3	M3	P4	TT	P5	m6	M6	M7	Uni	m2	M2	m3	M3	P4	TT	P5	m6	M6	m7	M7	Uni	m2	M2	m3	M3	P4	TT	P5
	Cent Dev.	5	10	15	20	25	30	35	40	45	-50	-45	-41	-36	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	24	29	34	39	44	49
19^x	Interval	m3	TT	M6	Uni	m3	TT	M6	Uni	m3	TT	M6	Uni	m3	TT	M6	Uni	m3	TT	M6	Uni	M2	P4	m6	M7	M2	P4	m6	M7	M2	P4
	Cent Dev.	-2	-5	-7	-10	-12	-15	-17	-20	-22	-25	-27	-30	-32	-35	-37	-40	-42	-45	-47	-50	48	45	43	40	38	35	33	30	28	25
23^x	Interval	TT	m2	P5	m2	P5	M2	m6	M2	M6	m3	M6	m3	m7	M3	m7	P4	M7	P4	M7	TT	Uni	TT	Uni	P5	m2	P5	M2	m6	M2	m6
	Cent Dev.	28	-43	-15	13	41	-30	-2	26	-46	-17	11	39	-32	-4	24	-48	-19	9	37	-35	-6	22	50	-21	7	35	-37	-8	20	48
29^x	Interval	m7	M6	P5	P4	m3	M2	Uni	m7	M6	P5	P4	M3	M2	Uni	m7	M6	P5	P4	M3	M2	Uni	M7	M6	P5	P4	M3	M2	Uni	M7	M6
	Cent Dev.	30	-41	-11	18	48	-23	7	37	-34	-4	25	-45	-15	14	44	-27	3	32	-38	-8	21	-49	-20	10	39	-31	-1	28	-42	-13
31^x	Interval	M7	M7	m7	m7	M6	M6	m6	m6	P5	TT	TT	P4	P4	M3	M3	m3	m3	M2	M2	m2	Uni	Uni	M7	M7	m7	m7	M6	M6	m6	m6
	Cent Dev.	45	-10	35	-20	25	-30	15	-40	5	50	-5	40	-15	30	-24	21	-34	11	-44	1	46	-9	36	-19	26	-29	16	-39	6	-49
37^x	Interval	m3	P4	m6	m7	m2	m3	TT	m6	M7	m2	M3	TT	M6	M7	M2	M3	P5	M6	Uni	M2	P4	P5	m7	Uni	m3	P4	m6	m7	m2	m3
	Cent Dev.	-49	3	-46	5	-43	8	-41	11	-38	13	-35	16	-33	19	-30	22	-27	24	-24	27	-22	30	-19	32	-16	35	-14	38	-11	40
41^x	Interval	M3	M6	m2	P4	M6	M2	TT	m7	m3	P5	M7	m3	m6	Uni	M3	M6	m2	P4	m7	M2	TT	m7	m3	P5	M7	M3	m6	Uni	M3	M6
	Cent Dev.	29	-42	-13	16	45	-26	3	32	-38	-9	20	49	-22	7	36	-35	-6	23	-48	-19	10	39	-32	-3	27	-44	-15	14	43	-28
43^x	Interval	P4	m7	m3	m6	M2	P5	Uni	P4	m7	m3	m6	m2	TT	Uni	P4	m7	m3	m6	m2	TT	M7	P4	m7	m3	m6	m2	TT	M7	M3	M6
	Cent Dev.	12	23	35	46	-42	-31	-19	-8	4	15	27	38	50	-39	-27	-16	-4	7	19	30	42	-47	-35	-24	-12	-1	11	22	34	46
46^x	Interval	P5	m2	m6	m3	M6	M3	M7	P4	Uni	P5	m2	m6	m3	M6	M3	m7	P4	Uni	TT	m2	m6	M2	M6	M3	m7	P4	Uni	TT	m2	m6
	Cent Dev.	-34	31	-3	-38	28	-7	-41	24	-10	-45	21	-14	-48	17	-17	48	14	-21	45	10	-24	41	7	-28	38	3	-31	34	0	-35
53^x	Interval	M6	P4	M2	M7	m6	M3	m2	m7	P5	m3	Uni	M6	TT	M2	M7	m6	M3	m2	m7	P5	m3	Uni	M6	TT	M2	M7	m6	P4	m2	m7
	Cent Dev.	-26	47	21	-6	-32	41	15	-12	-38	35	9	-18	-44	29	3	-24	50	23	-3	-30	44	17	-9	-36	38	11	-15	-42	32	5
59^x	Interval	M7	M6	m6	TT	P4	M3	M2	m2	M7	m7	M6	P5	TT	M3	m3	m2	Uni	M7	M6	m6	TT	P4	M3	M2	m2	M7	m7	M6	P5	TT
	Cent Dev.	-41	18	-22	37	-4	-45	14	-27	33	-8	-49	10	-31	28	-12	47	6	-35	24	-17	43	2	-39	20	-21	38	-2	-43	16	-25
61^x	Interval	M7	m7	m7	M6	m6	P5	TT	P4	P4	M3	m3	M2	m2	Uni	Uni	M7	m7	M6	m6	P5	P5	TT	P4	M3	m3	M2	M2	m2	Uni	M7
	Cent Dev.	17	34	-49	-32	-16	1	18	35	-48	-31	-14	3	20	36	-47	-30	-13	4	21	38	-45	-29	-12	5	22	39	-44	-27	-10	7

All Primes Raised to the First Power Comprise Part II

Commas
for Ed Carroll

Mike Winter (2005)



-Empty Page-

Commas
for Ed Carroll

1st Realization Addendum

Flute
Oboe
Clarinet in B♭
Bass Clarinet in B♭
Bassoon
Trumpet in C
Quartertone Flugelhorn in B♭
Horn in F
Trombone
Harp
2 Digital Pianos (controlling a software sampler)
2 Percussionists (playing any pitched percussion instruments)
2 Violins
Viola
2 Cellos
Contrabass

For the first performance of Commas for Ed Carroll by the California Institute of the Arts New Century Players, I have arranged the work to best suite the ensemble. Because of logistical considerations of the concert, a software sampler is implemented to simulate several retuned pianos. This has been made possible by a generous donation from the software development company, Native Instruments.

Included in this addendum, is a master score that details the arrangements made for this performance and an abbreviated version of the master notes. Upon request, I can make available transposed parts that give each performer an extended available pitch set for the segments he or she is responsible for with the approximate playing times of the given pitch sets.

There are not separate parts for the Harpist and the Percussionists since they are playing C's throughout. They should only need the master score. The parts for the Pianists document the pitch-to-key mappings of the samples for each segment or texture.

To the Performers

Thank you so much for your participation in this performance. I would especially like to thank Ed Carroll for his continual support and the encouragement he gave that led to the writing of this piece.

This work is meant to be an exploration of commas. I have included the possibility for infinite pitch sets to facilitate such exploration. The most important aspect of the work is that you, the performers, attempt to hear and play these possibly unfamiliar, exotic

intervals out of sheer curiosity. Though the process that went into determining the infinite pitch sets is very strict, it still allows for the performers to explore in their own ways. Do not feel pressured to play all the notes. Just play as many as you can as accurately as possible.

Because of the arrangements that have been made with the software sampler, all performers have a steady reference of pitch on stage. Basically, if you are in tune with one of the pianos, you are playing a correct pitch.

Commas
for Ed Carroll

Abbreviated Notes

When a prime number is raised to any power greater than zero it will never equal another prime number raised by any power; however, it may come very close. These near misses are called commas and are the phenomena explored in this work. The fact that commas exist is one of the fundamental issues of tuning theory. One well-documented comma is the Pythagorean comma, which is the interval created when twelve stacked perfect fifths come close to seven stacked octaves. In theory, there are an infinite number of commas.

Notation

The score is composed of two textures labeled Texture I and Texture II. Texture II is a set of available pitches that are played throughout the piece and Texture I is a sequence of segments with pitches that are determined by primes raised to powers.

Above each note is a deviation in cents, one-hundredth of a semitone, from the closest written pitch in a twelve-tone equal-tempered tuning system. Below each note is a harmonic number, which represents the position of that pitch in the harmonic series if it was not reduced into the octave of the reference fundamental, represented by the under-number 1. Even though the pitches are displayed in one octave they may be transposed by octaves in either direction, e.g. the written pitch represents a pitch class that should be played within the pitch range determined by the lower graph in the score (See ***Pitch Range***).

Texture I

Each segment, represented by a measure, contains available pitches based on stacked intervals, which are then reduced and displayed in one octave. Each successive segment overlaps the previous segment by fifteen seconds. Players should attempt to play pitches that are close to pitches that are sounding from the Texture II players.

The under-numbers in Texture I are the result of the expression $\text{Prime}^{x}\}_{x\geq 0}^{7}$.

In the score, each segment displays x from zero to the seventh degree, but the available pitch set could include pitches that extend the values of x to a much greater degree and can theoretically extend infinitely;
$\text{Prime}^{x}\}_{x\geq 0}^{\infty}$.

A more extensive list of the possible pitches for each segment is given on the second page of the score.

Once a segment begins, all written pitches in musical notation and the pitches that can be determined by the process described in the main notes can be played till the end of that segment in any order. The linear documentation on the page should not necessarily be interpreted to suggest that those pitches be played in succession.

Texture II

The pitches written in Texture II are played throughout the entire piece. They are the primes raised to the first power. Players should try to play pitches that are not already sounding.

Ranges of Tone Durations and Dynamics

In the middle of the score is a graph that illustrates the possibilities of tone durations and dynamics. The boundaries off the gray area show the limits of possible tone durations and dynamics. For tone durations, the y-axis is linear from 0 to 3 seconds, but for dynamics it is inverted. Of utmost importance is that the middle of each successive segment should get louder with shorter tone duration creating a progression of increasing density and higher subjective intensity. During the overlapping portions of the segments the tone durations and dynamics should always retreat to between 1 and 3 seconds and between *pianissimo* and *mezzo piano*, respectively.

Pitch Ranges

At the bottom of the score is a graphic representation of the pitch ranges (illustrated by the gray area) that the ensemble should play within. The piece should start in the middle of the ensemble's range with a span of approximately one octave (C4 to C5 for this realization). By the last segment before the middle section the ensemble should be playing throughout the ensemble's entire range (C1 to C8).

This Realization

The ensemble should start by playing Texture II before starting Texture I. Between Section I and the retrograde is a middle section with three combinations of segments from Section I. These are documented by the prime numbers under the measures on the second page. All the performers should play Texture II when they are not responsible for a segment in Texture I.

Commas

for Ed Carroll

Mike Winter (2005)

Score of 1st Realization

Note

*

All performers play Texture II during times when they are not responsible for a segment in Texture I.

*

Percussionists may play any pitched percussion instruments bowed or normal.

*

The harpist and the percussionists only play Cs throughout.

*

Texture I
1 to 3 Voices
Piano I
Throughout

ca. 45 sec. | ca. 1 min. | ca. 30. sec | ca. 1 min. | ca. 30 sec. | ca. 1 min. | ca. 30 sec. | ca. 1 min. | ca. 30 sec. | ca. 30 sec.

+0 –14 –27 –41 +45 +32 +18 +4 — 1 5 25 — **5x** Violin II

+0 –49 +3 –46 +5 –43 +8 –41 — 1 11 121 — **11x** Flugelhorn

+0 +5 +10 +15 +20 +25 +30 +35 — 1 17 289 — **17x** Flute, Trombone

+0 +28 –43 –15 +13 +41 –30 –2 — 1 23 529 — **23x** Clarinet, Bass Clarinet, Viola

+0 +45 –10 +35 –20 +25 –30 +15 — 1 31 961 — **31x** Flugelhorn, Violin I, Contrabass

+0 +2 +4 +6 +8 +10 +12 +14 — 1 3 9 — **3x** Clarinet

+0 –31 +38 +6 –25 +44 +13 –18 — 1 7 49 — **7x** Horn

+0 +41 –19 +22 –38 +3 +43 –16 — 1 13 169 — **13x** Violin I, Cello II

+0 –2 –5 –7 –10 –12 –15 –17 — 1 19 361 — **19x** Oboe, Bassoon

+0 +30 –41 –11 +18 +48 –23 +7 — 1 29 841 — **29x** Trumpet, Violin II, Cello I

Move to Middle Section

t = current time (in seconds)

s = number of segments

Upper Limit = 3

$$\textit{Lower Limit} = 1 + \frac{t\left(-.5 + .5\,Cos\left(1 - \frac{2\pi t}{45}\right)\right)}{15(-1+3s)} \quad t \geq 0 \quad t = (\text{Total Time of Section 1})$$

pp 3" — *p* 2" — *mp* 1" — *mf* 0"

ca. 1 min. | ca. 30 sec. | ca. 1 min. | ca. 30 sec. | ca. 1 min. | ca. 30 sec. | ca. 1 min. | ca. 30 sec. | ca. 1 min. | ca. 15"

+0 +2 –14 –31 –49 +41 +5 –2 +28 +30 +45

1 3 5 7 11 13 17 19 23 29 31

Texture II
# of Primes ≥ # of Voices ≥ # of Primes / 2
Piano II Throughout

Section 1

Retrograde

Upper Limit of Ensemble's Range — C8

Middle of Ensemble's Range — C5 F#5 C6 F#6 C7 F#7

Middle of Ensemble's Range — C4 F#3 C3 F#2 C2 F#1 C1

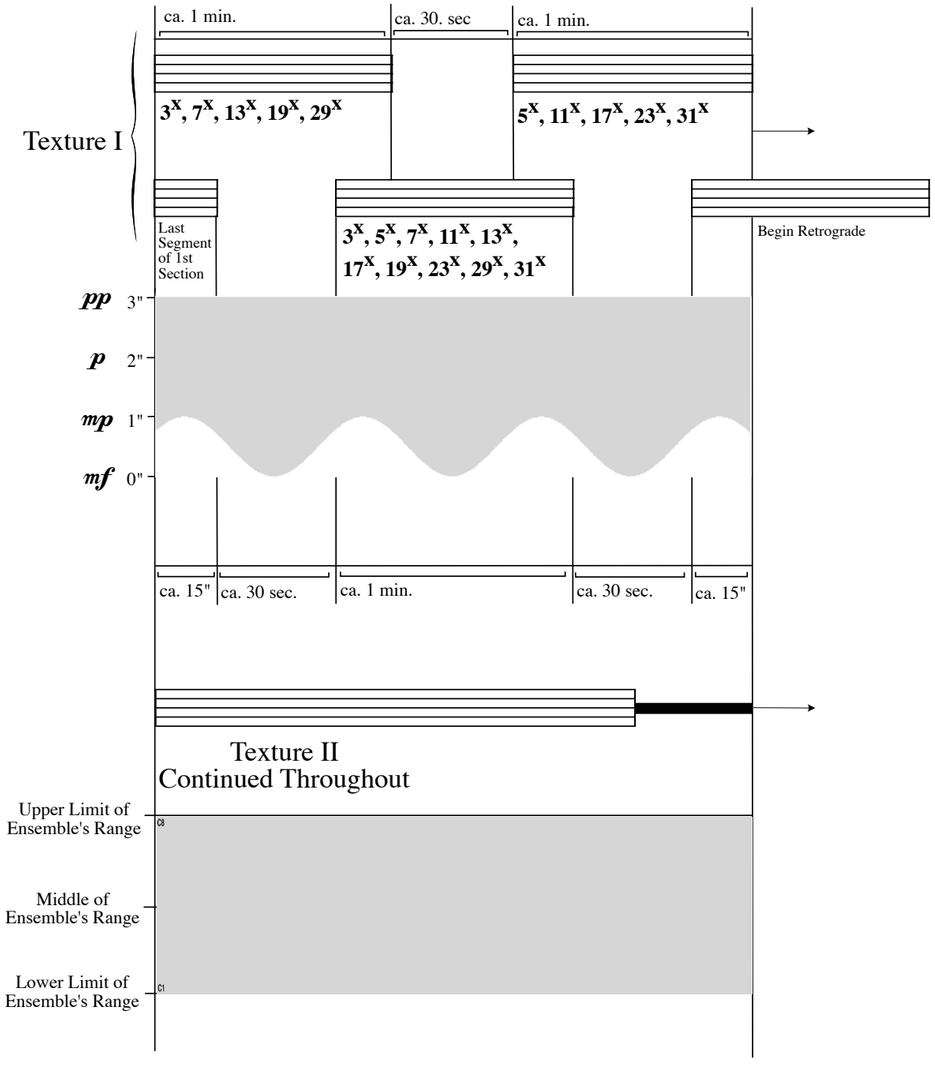
Lower Limit of Ensemble's Range

Commas

for Ed Carroll

Mike Winter (2005)

Middle Section



More Extensive List of Possible Pitches for Each Segment

x		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
3^x	Interval	P5	M2	M6	M3	M7	TT	m2	m6	m3	m7	P4	Uni	P5	M2	M6	M3	M7	TT	m2	m6	m3	m7	P4	Uni	P5	m3	m7	P4	Uni	P5
	Cent Dev.	2	4	6	8	10	12	14	16	18	20	22	23	25	27	29	31	33	35	37	39	41	43	45	47	49	-49	-47	-45	-43	-41
5^x	Interval	M3	m6	Uni	m3	P5	M7	m3	P5	M7	m3	TT	m7	M2	TT	m7	M2	TT	m7	m2	P4	M6	m2	P4	M6	m2	M3	m6	Uni	M3	m6
	Cent Dev.	-14	-27	-41	45	32	18	4	-9	-23	-37	49	36	22	8	-5	-19	-33	-46	40	26	13	-1	-15	-28	-42	44	30	17	3	-11
7^x	Interval	m7	P5	P4	m3	Uni	m7	m6	TT	m3	m2	M7	m6	TT	M3	m2	M7	M6	TT	M3	M2	M7	M6	P5	P4	M2	Uni	m7	P5	P4	m3
	Cent Dev.	-31	38	6	-25	44	13	-18	-49	19	-12	-43	26	-5	-36	32	1	-30	39	8	-23	45	14	-17	-48	21	-11	-42	27	-4	-35
11^x	Interval	TT	M7	P4	m7	M3	M6	m3	m6	M2	P5	m2	TT	Uni	P4	M7	M3	m7	m3	M6	M2	m6	m2	P5	Uni	TT	M7	P4	m7	M3	M6
	Cent Dev.	-49	3	-46	5	-43	8	-41	11	-38	13	-36	16	-33	18	-30	21	-28	24	-25	26	-22	29	-20	32	-17	34	-14	37	-12	40
13^x	Interval	m6	P4	m2	m7	TT	M2	M7	P5	M3	Uni	m6	P4	m2	m7	TT	M2	M7	P5	M3	Uni	M6	P4	m2	m7	TT	m3	M7	P5	M3	Uni
	Cent Dev.	41	-19	22	-38	3	43	-16	24	-35	5	46	-14	27	-33	8	48	-11	29	-30	11	-49	-8	32	-27	13	-46	-6	35	-25	16
17^x	Interval	m2	M2	m3	M3	P4	TT	P5	m6	M6	M7	Uni	m2	M2	m3	M3	P4	TT	P5	m6	M6	m7	M7	Uni	m2	M2	m3	M3	P4	TT	P5
	Cent Dev.	5	10	15	20	25	30	35	40	45	-50	-45	-41	-36	-31	-26	-21	-16	-11	-6	-1	4	9	14	19	24	29	34	39	44	49
19^x	Interval	m3	TT	M6	Uni	m3	TT	M6	Uni	m3	TT	M6	Uni	m3	TT	M6	Uni	m3	TT	M6	Uni	M2	P4	m6	M7	M2	P4	m6	M7	M2	P4
	Cent Dev.	-2	-5	-7	-10	-12	-15	-17	-20	-22	-25	-27	-30	-32	-35	-37	-40	-42	-45	-47	-50	48	45	43	40	38	35	33	30	28	25
23^x	Interval	TT	m2	P5	m2	P5	M2	m6	M2	M6	m3	M6	m3	m7	M3	m7	P4	M7	P4	M7	TT	Uni	TT	Uni	P5	m2	P5	M2	m6	M2	m6
	Cent Dev.	28	-43	-15	13	41	-30	-2	26	-46	-17	11	39	-32	-4	24	-48	-19	9	37	-35	-6	22	50	-21	7	35	-37	-8	20	48
29^x	Interval	m7	M6	P5	P4	m3	M2	Uni	m7	M6	P5	P4	M3	M2	Uni	m7	M6	P5	P4	M3	M2	Uni	M7	M6	P5	P4	M3	M2	Uni	M7	M6
	Cent Dev.	30	-41	-11	18	48	-23	7	37	-34	-4	25	-45	-15	14	44	-27	3	32	-38	-8	21	-49	-20	10	39	-31	-1	28	-42	-13
31^x	Interval	M7	M7	m7	m7	M6	M6	m6	m6	P5	TT	TT	P4	P4	M3	M3	m3	m3	M2	M2	m2	Uni	Uni	M7	M7	m7	m7	M6	M6	m6	m6
	Cent Dev.	45	-10	35	-20	25	-30	15	-40	5	50	-5	40	-15	30	-24	21	-34	11	-44	1	46	-9	36	-19	26	-29	16	-39	6	-49
37^x	Interval	m3	P4	m6	m7	m2	m3	TT	m6	M7	m2	M3	TT	M6	M7	M2	M3	P5	M6	Uni	M2	P4	P5	m7	Uni	m3	P4	m6	m7	m2	m3
	Cent Dev.	-49	3	-46	5	-43	8	-41	11	-38	13	-35	16	-33	19	-30	22	-27	24	-24	27	-22	30	-19	32	-16	35	-14	38	-11	40
41^x	Interval	M3	M6	m2	P4	M6	M2	TT	m7	m3	P5	M7	m3	m6	Uni	M3	M6	m2	P4	m7	M2	TT	m7	m3	P5	M7	M3	m6	Uni	M3	M6
	Cent Dev.	29	-42	-13	16	45	-26	3	32	-38	-9	20	49	-22	7	36	-35	-6	23	-48	-19	10	39	-32	-3	27	-44	-15	14	43	-28
43^x	Interval	P4	m7	m3	m6	M2	P5	Uni	P4	m7	m3	m6	m2	TT	Uni	P4	m7	m3	m6	m2	TT	M7	P4	m7	m3	m6	m2	TT	M7	M3	M6
	Cent Dev.	12	23	35	46	-42	-31	-19	-8	4	15	27	38	50	-39	-27	-16	-4	7	19	30	42	-47	-35	-24	-12	-1	11	22	34	46
46^x	Interval	P5	m2	m6	m3	M6	M3	M7	P4	Uni	P5	m2	m6	m3	M6	M3	m7	P4	Uni	TT	m2	m6	M2	M6	M3	m7	P4	Uni	TT	m2	m6
	Cent Dev.	-34	31	-3	-38	28	-7	-41	24	-10	-45	21	-14	-48	17	-17	48	14	-21	45	10	-24	41	7	-28	38	3	-31	34	0	-35
53^x	Interval	M6	P4	M2	M7	m6	M3	m2	m7	P5	m3	Uni	M6	TT	M2	M7	m6	M3	m2	m7	P5	m3	Uni	M6	TT	M2	M7	m6	P4	m2	m7
	Cent Dev.	-26	47	21	-6	-32	41	15	-12	-38	35	9	-18	-44	29	3	-24	50	23	-3	-30	44	17	-9	-36	38	11	-15	-42	32	5
59^x	Interval	M7	M6	m6	TT	P4	M3	M2	m2	M7	m7	M6	P5	TT	M3	m3	m2	Uni	M7	M6	m6	TT	P4	M3	M2	m2	M7	m7	M6	P5	TT
	Cent Dev.	-41	18	-22	37	-4	-45	14	-27	33	-8	-49	10	-31	28	-12	47	6	-35	24	-17	43	2	-39	20	-21	38	-2	-43	16	-25
61^x	Interval	M7	m7	m7	M6	m6	P5	TT	P4	P4	M3	m3	M2	m2	Uni	Uni	M7	m7	M6	m6	P5	P5	TT	P4	M3	m3	M2	M2	m2	Uni	M7
	Cent Dev.	17	34	-49	-32	-16	1	18	35	-48	-31	-14	3	20	36	-47	-30	-13	4	21	38	-45	-29	-12	5	22	39	-44	-27	-10	7

All Primes Raised to the First Power Comprise Part II